J. Fluid Mech. (2005), vol. 538, pp. 189–198. © 2005 Cambridge University Press
doi:10.1017/S0022112005005549 Printed in the United Kingdom

Fluctuating circulation forced by unsteady multidirectional breaking waves

By A. B. KENNEDY

Department of Civil & Coastal Engineering, University of Florida, Gainesville, FL 32611-6590, USA

(Received 11 July 2004 and in revised form 24 May 2005)

In this paper, we consider nearshore rotational currents directly forced by unsteady multidirectional wave breaking. Scaling relationships, simplified analytical solutions, and asymptotic limits are developed for the maximum forced cross-shore and longshore velocities. In all cases, forced longshore velocities are considerably larger than cross-shore velocities. On longshore-uniform beaches, strong fluctuating velocities are found for (i) large waves; (ii) strong directional spreading; and (iii) short peak wave periods. When topographic inhomogeneities control longshore scales of wave breaking, overall scaling changes and the largest fluctuating velocities are found for (*a*) large waves; (*b*) long wave periods; and (*c*) topographic features that vary quickly in the longshore direction. The ratio of fluctuating rotational velocities to mean longshore current does not depend on the wave height or period, but instead on the bottom friction, slope, deep water wave angle, and details of the wave spectrum.

1. Introduction

Even casual observation of waves at a beach reveals that breaking is unsteady in both space and time. This unsteady breaking, which is tied to short-wave groups, generates both irrotational low-frequency waves and rotational low-frequency circulation. While there has been considerable study of irrotational low-frequency waves (Longuet-Higgins & Stewart 1962; Foda & Mei 1981; many others), there has been very little work on rotational motions generated by unsteady breaking waves, which have never even received a basic scaling analysis. This is despite the presumed importance of rotational motions in phenomena such as dangerous 'migrating rip currents' (Fowler & Dalrymple 1990), and large-scale Reynolds stresses which will affect the mean current profile.

This neglect of rotational motions is partly because of the difficulty in measuring instantaneous vorticity using a sparse array of fixed instruments, but is also because the widely used theory of radiation stresses (Longuet-Higgins & Stewart 1964) makes it difficult to separate rotational forcing from irrotational forcing. However, recent theoretical developments have demonstrated that dissipation-based forcing, which is tied to the rate of generation of circulation (Peregrine 1998, 1999; Bühler & Jacobson 2001; Brocchini *et al.* 2004), is very useful in estimating the available rotational forcing.

In this paper, we develop scaling relations and simplified analytical solutions for nearshore rotational motions forced by breaking wave groups. Dimensional examples show that the strength of these motions varies strongly depending on characteristics of the incident wave spectrum and can range from insignificant to strong.

2. Scaling for circulation generated by unsteady breaking waves

Over an unsteady circulation cell, scaling for the rotational velocities forced directly by unsteady breaking waves has three components: a rate of generation of circulation, $(\mathrm{D}\Gamma/\mathrm{D}t)_0$ (where the circulation $\Gamma \equiv \oint(u, v)\mathrm{d}l$), a forcing frequency, Δf, and a length scale which may be different for cross-shore and longshore motions, L_0. Given these, the velocity scaling is

$$O(u, v) = O\left(\left(\frac{\mathrm{D}\Gamma}{\mathrm{D}t}\right)_0 \frac{1}{\Delta f L_0}\right) \tag{2.1}$$

where (u, v) are the rotational fluctuating cross-shore and longshore velocities, respectively. Characteristic time scales may be linked directly to the incident wave spectrum by replacing the infinite array of second-order subharmonic frequencies with one characteristic frequency

$$O(\Delta f) = O(\delta_f f_p) \tag{2.2}$$

where f_p is the peak wave frequency, and δ_f is presumed to be small. Thus, a narrow swell spectrum would have a low value of δ_f when compared to a more broad-banded wind sea.

For waves breaking all the way to the shoreline, scaling for the maximum rate of generation of circulation can be found from Brocchini *et al.* (2004, equation (2.21) or (2.23)) as

$$O\left(\left(\frac{\mathrm{D}\Gamma}{\mathrm{D}t}\right)_0\right) = O(gH_b), \tag{2.3}$$

where H_b is a representative breaking wave height.

The characteristic length scale, L_0, may be different for the cross-shore and longshore coordinates in much the same way that horizontal and vertical velocity scales vary for linear waves in shallow water. This scale will also change if the beach topography changes from longshore-uniform to longshore-varying. These differences are significant and can change strongly the system response.

2.1. *Longshore-uniform beaches*

On a longshore-uniform beach, all length scales parallel to the shoreline are determined entirely from the incident wave spectrum. A representative longshore wavenumber for waves with a deep water mean direction of θ_0 from the shore normal may be found from geometry and the deep water dispersion relation $k_0 = f_p^2/g$ as

$$O(\Delta k) = O\left(\delta_k \cos\theta_0 \frac{f_p^2}{g}\right) \tag{2.4}$$

where δ_k is a presumed small number which represents the directional width of the wave spectrum. From Snell's law, longshore wavenumbers on a longshore-uniform beach are conserved, so this scaling remains valid in all depths.

If longshore and cross-shore scales are the same, then the velocity scaling becomes

$$\begin{aligned} O(u, v) &= O\left(\frac{gH_b\Delta k}{\delta_f f_p}\right) \\ &= O\left(H_b f_p \frac{\delta_k \cos\theta_0}{\delta_f}\right), \end{aligned} \tag{2.5}$$

If longshore distances $2\pi/\Delta k$ are large when compared to the cross-shore distance x_c, which scales with surf zone width ($\Delta k x_c \ll 1$), then boundary-layer-type scaling applies for the cross-shore velocity, u

$$O(u) = O(\Delta k x_c v) = O\left(\frac{H_b^2 f_p^3}{gm}\frac{\delta_k^2 \cos^2\theta_0}{\delta_f}\right), \tag{2.6}$$

where m is a representative bottom slope so that $O(h) = O(mx)$, and the scaling for v remains the same. As we will see, this scaling for u is applicable to many typical cases, although there are instances where cross-shore and longshore scales are comparable and (2.5) is more appropriate.

Thus, rotational velocities on longshore-uniform beaches forced by unsteady multidirectional waves are predicted to be most significant for

(*a*) large waves,
(*b*) a large deep water directional width compared to spectral width,
(*c*) mean wave directions close to the shore normal, and
(*d*) higher frequency wind waves.

The first three items might be expected, but the fourth is a surprise. Rotational velocities on open beaches are larger for higher frequency waves because longshore group length scales increase with the square of the wave period but group frequencies decrease only with the inverse of the wave period. A notable omission from (2.5) is the beach slope m: fluctuating longshore velocities are quite insensitive to slope, while for small kx_c, (2.6) predicts the cross-shore velocity to decrease with increasing slope.

Local storm waves, which often have relatively small periods, wide directional and frequency distribution, and large heights, thus appear likely to have large fluctuating velocities. Bi-directional spectra, which may have a large directional width with a relatively narrow frequency spectrum, are predicted to have large velocities. Swell waves, with low spreading and peak frequencies, are not. For small Δkx_c (large longshore length scales), directly forced cross-shore velocities appear to be small.

2.2. *Bathymetric non-uniformities*

On many beaches, there are strong bathymetric non-uniformities in the longshore direction with typical length scale $2\pi/k_t$. If bathymetric length scales are small compared to the longshore wave group length scales, i.e. $k_t \gg \delta_k \cos\theta_0 k_0$, then these topographic variations will control the length scales of wave breaking. A typical example would be a topographically controlled rip current, where the rip channel width is often small compared to longshore wave group scales.

This can result in large changes in unsteady velocity scaling when compared to the longshore-uniform case. Using this new length scale, which is assumed to be small enough that longshore and cross-shore velocity scales are similar,

$$O(u, v) = O\left(\frac{gH_b k_t}{\delta_f f_p}\right). \tag{2.7}$$

Thus when topographic length scales dominate, unsteady velocity fluctuations increase with increasing wave group period (decreasing wave group frequency). This is opposite from what was found on longshore-uniform beaches; the underlying reason for this great difference is that length scales on longshore-uniform beaches change with wave period, while the scales on longshore-varying topographies are fixed. A similar result for longshore-varying topographies was previously found by Kennedy (2003) using a simple discrete vortex model, and agrees with anecdotal evidence that topographic rip currents are more dangerous with longer period swell waves (Lascody 1998).

3. Analytic flow field for longshore-periodic vorticity

To complement and test the scaling relations of the previous section, it is useful to develop a simplified solution which can give the rotational velocity field for forcing with specified length and time scales. Here, we develop the simplest possible solution: a planar beach with depth $h = mx$, where $h_{,x} = m$ is the beach slope, $h_{,y} = 0$, and periodic longshore velocities. This is quite obviously approximate, much like the triangular longshore current of Longuet-Higgins (1970*a*). Still, it provides a good first step towards understanding the kinematics of rotational nearshore flows. Solutions are similar in character to the steady rip current solution for linear bottom friction given by Bowen (1969) and also have some similarity with additional solutions in Bowen's (1967) thesis†, although details are different.

We define a transport stream function ψ so that

$$u = -\frac{\psi_{,y}}{h}, \quad v = \frac{\psi_{,x}}{h} \tag{3.1}$$

where h is the depth and $_{,(-)}$ denotes differentiation in the (—)-coordinate.

For a solution which is periodic in y with wavenumber k, so that $\psi = f(x)/2 \exp \mathrm{i}ky$ + c.c., the relation on a planar beach with vorticity $\omega \equiv v_{,x} - u_{,y} = \omega_0/2 \exp \mathrm{i}ky$ + c.c. then becomes

$$xf_{,xx} - f_{,x} - k^2xf = m\omega_0 x^2. \tag{3.2}$$

The general solution to this is

$$f = C_1 x I_1(kx) + C_2 x K_1(kx) + \frac{m\pi\omega_0}{2k^2} x L_1(kx) \tag{3.3}$$

where C_1 and C_2 are arbitrary constants, I_1 and K_1 are modified Bessel functions, and L_1 is a modified Struve function (Abramowitz & Stegun 1964, ch. 9, 12).

In this form, the solution is not immediately useful, as no flows are possible with vorticity in the nearshore and no vorticity in deeper water. To overcome this, we divide into two regions: $0 \leqslant x \leqslant x_c$ with vorticity $\omega = \omega_0/2 \exp \mathrm{i}ky$ + c.c.; and $x > x_c$ with zero vorticity. By employing matching conditions for the stream functions and velocities at the boundary $x = x_c$, and ensuring that the solution goes to $\psi = 0$ at $x = 0$ and $x = \infty$ (zero flow through onshore or offshore boundaries, or between cells), the stream function becomes

$$\left.\begin{aligned} \psi &= \tfrac{1}{2}\left[C_1 x I_1(kx) + \frac{m\omega_0\pi}{2k^2} x L_1(kx)\right] \exp \mathrm{i}ky + \text{c.c.}, & x \leqslant x_c, \\ \psi &= \tfrac{1}{2} C_2 x K_1(kx) \exp \mathrm{i}ky + \text{c.c.}, & x > x_c, \end{aligned}\right\} \tag{3.4}$$

where

$$(C_1, C_2) = \frac{m x_c \omega_0 \pi}{2k}(A_1, A_2) \tag{3.5}$$

and

$$\left.\begin{aligned} A_1 &= -L_1(kx_c)K_0(kx_c) - L_0(kx_c)K_1(kx_c), \\ A_2 &= L_1(kx_c)I_0(kx_c) - L_0(kx_c)I_1(kx_c), \end{aligned}\right\} \tag{3.6}$$

† Thanks to an anonymous referee for pointing this out.

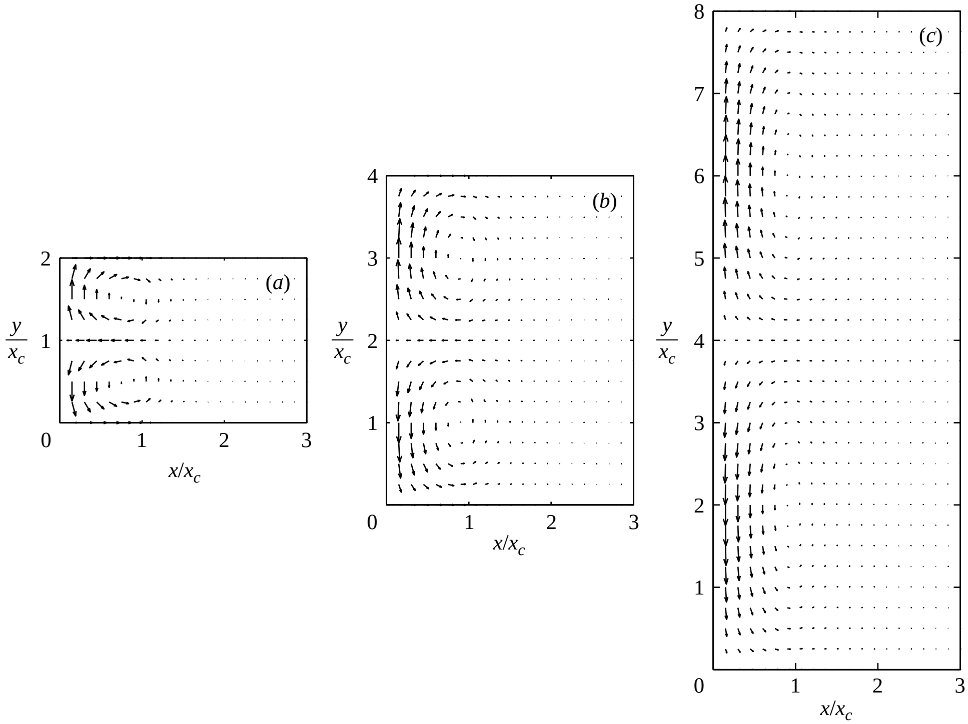

FIGURE 1. Undistorted velocity field of arbitrary amplitude caused by the longshore-periodic generation of vorticity. (*a*) $kx_c = \pi$; (*b*) $kx_c = \pi/2$; (*c*) $kx_c = \pi/4$.

The velocities can then be found as

$$\left.\begin{aligned} u &= -\frac{\mathrm{i}}{2}\left[C_1 \frac{k}{m} I_1(kx) + \frac{\omega_0 \pi}{2k} L_1(kx)\right] \exp \mathrm{i}ky + \text{c.c.}, && x \leqslant x_c, \\ u &= -\frac{\mathrm{i}}{2} C_2 \frac{k}{m} K_1(kx) \exp \mathrm{i}ky + \text{c.c.}, && x > x_c, \end{aligned}\right\} \tag{3.7}$$

and

$$\left.\begin{aligned} v &= \tfrac{1}{2}\left[C_1 \frac{k}{m} I_0(kx) + \frac{\pi\omega_0}{2k} L_0(kx)\right] \exp \mathrm{i}ky + \text{c.c.}, && x \leqslant x_c, \\ v &= -\tfrac{1}{2} C_2 \frac{k}{m} K_0(kx) \exp \mathrm{i}ky + \text{c.c.}, && x > x_c, \end{aligned}\right\} \tag{3.8}$$

This solution makes use of identities for derivatives of modified Bessel functions and modified Struve functions as found in Abramowitz & Stegun (1964).

Figure 1 gives velocity fields for three different aspect ratios. Two properties appear most striking: (i) longshore velocities dominate over cross-shore velocities as kx_c becomes small; and (ii) longshore velocities shoreward of $x = x_c$ are much stronger than velocities seaward of $x = x_c$ for small kx_c. The first was predicted in (2.6) from boundary layer arguments but the strong relative decrease in u remains notable. The second again might be expected mathematically, but the concentration of velocities toward the shoreline for $kx_c \ll 1$ is very strong. Qualitatively, these also resemble the steady rip currents given by Bowen (1969), which used different assumptions. Thus, the presence of the shoreline appears to provide a strong constraint on the form of any nearshore circulation cell, regardless of the details.

The total circulation over one half of a circulation cell pair may be found from Green's theorem as

$$\Gamma \equiv \iint \omega \, \mathrm{d}A \tag{3.9}$$

where ω is the vorticity and $\mathrm{d}A$ represents integration over the circulation cell area, and thus

$$\omega_0 = \frac{\Gamma k}{2x_c}. \tag{3.10}$$

When longshore length scales are long compared to cross-shore scales ($kx_c \ll 1$), the modified Bessel and Struve functions can be simplified (Abramowitz & Stegun 1964) so that maximum velocities become

$$u = \frac{3\Gamma k}{32}(kx_c), \quad v = \frac{\Gamma k}{2}. \tag{3.11}$$

These have $O(u/v) = O(kx_c)$ and thus agree with the scaling relations in § 2 for small kx_c. Maximum cross-shore velocities are found at $x = 3x_c/4$ and maximum longshore velocities are found at $x = 0$.

3.1. *Alternative analytical solution*

Details of the velocity field described above depend on the distribution of vorticity, which was determined fairly crudely. Because of this, we offer another analytical solution to test the sensitivity of maximum velocities to the detailed vorticity distribution. The alternative solution has all vorticity concentrated in a delta function at $x = x_c$ so that $\omega = \omega_0/2\delta(x - x_c)\exp(iky) + \text{c.c.}$, where $\delta(x - x_c)$ is the Dirac delta function, and has a stream function given by

$$\left.\begin{aligned} \psi &= \tfrac{1}{2}C_3 x K_1(kx_c) I_1(kx) \exp \mathrm{i}ky + \text{c.c.}, \quad x \leqslant x_c, \\ \psi &= \tfrac{1}{2}C_3 x I_1(kx_c) K_1(kx) \exp iky + \text{c.c.}, \quad x > x_c, \end{aligned}\right\} \tag{3.12}$$

where

$$C_3 = \frac{\Gamma m k x_c}{2}. \tag{3.13}$$

For any flow field, the maximum longshore and cross-shore velocities always occur at $x = x_c$, and are given as

$$\left.\begin{aligned} u_{max} &= \tfrac{1}{2}kx_c \Gamma k K_1(kx_c) I_1(kx_c), \\ v_{max} &= \tfrac{1}{2}kx_c \Gamma k K_1(kx_c) I_0(kx_c), \end{aligned}\right\} \tag{3.14}$$

These locations of maximum velocity differ significantly from the previous section. Thus, it would seem that the location of maximum fluctuating velocities may difficult to predict accurately without a more sophisticated model.

For $kx_c \ll 1$, asymptotic maximum velocities are

$$u = \frac{\Gamma k}{2}\frac{kx_c}{2}, \quad v = \frac{\Gamma k}{2}. \tag{3.15}$$

This asymptotic longshore velocity v is the same as found in (3.11), but the cross-shore velocity u in (3.15) is much greater. Thus, it appears that the magnitude of the fluctuating longshore velocity is less sensitive to the detailed vorticity distribution than the cross-shore velocity.

3.2. *Dimensional examples*

Scaling relationships may be tested against the analytical velocity field using typical dimensional quantities. In this way, we may determine situations in which unsteady

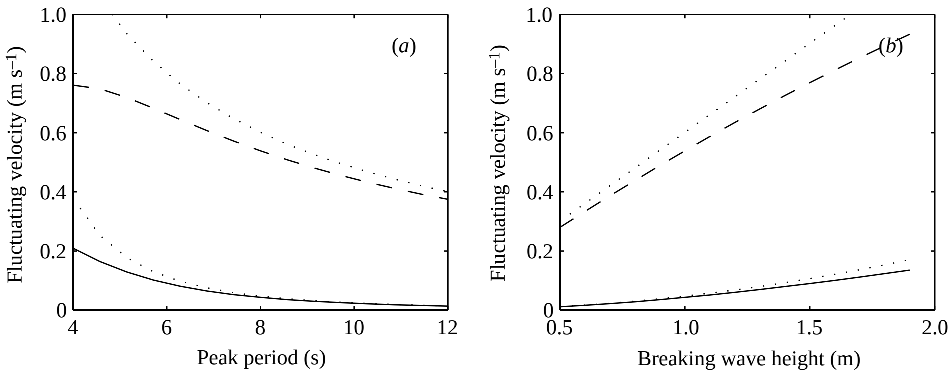

FIGURE 2. Maximum longshore (——) and cross-shore (—) velocities from (3.7)–(3.8); and asymptotic limits (···) (3.11) from unsteady multidirectional wave forcing using a simplified two-component model with circulation calculated from (3.16)–(3.17). Base case: $T_p = 8$ s, $m = 0.03$, $\Delta k = 0.2\kappa_0$, $\theta_0 = 0$, $\Delta f = 0.2 f_p$, $H_b = 1$ m, $\gamma = 0.78$. (*a*) Changing peak period; (*b*) changing breaking wave height.

multidirectional waves are likely to cause significant rotational velocity fluctuations.

In line with the simplified analytical representation of flow and with the scaling, we represent an incident wave spectrum using a single subharmonic wavenumber $\Delta k = \delta_k k_0 \cos\theta_0$ and frequency $\Delta f = \delta_f f_p$. The rotational response at this wavenumber/frequency pair is taken as representative of the entire spectrum. Of course this is a gross simplification but, because there are no leading-order resonances which might concentrate energy at preferred wavenumber-frequency combinations, it should provide a useful estimate of the directly forced fluctuating velocity field.

This solution may be an upper bound on fluctuating velocities for two reasons: (i) bottom friction is not included; and (ii) the fluctuating rate of generation of circulation will be assumed to have an amplitude equal to the mean rate of generation of circulation, $(\mathrm{D}\Gamma/\mathrm{D}t)_0$, and to not vary in phase through the surf zone. Accounting for these would reduce velocities, but including them in a simple analytical solution is beyond the scope of this paper.

We estimate the rate of generation of circulation from Brocchini *et al.* (2004). For waves breaking on a planar beach through to the shoreline, type (ii) breaking ($H = \gamma h$) appears the most appropriate estimate:

$$\frac{\mathrm{D}\Gamma}{\mathrm{D}t_0} = \frac{5g}{16}\gamma H_b \tag{3.16}$$

where γ is the ratio of wave height to water depth at breaking, $\gamma = H/h$. The fluctuating rate of generation of circulation is thus taken as

$$\frac{\mathrm{D}\Gamma}{Dt} = \frac{1}{2}\frac{\mathrm{D}\Gamma}{Dt_0}\exp\mathrm{i}(\Delta k y + 2\pi\Delta f t) + \mathrm{c.c.} \tag{3.17}$$

This has the form of a progressive wave, but maximum velocities are identical if the forcing is taken to be a standing wave.

The base case here is taken as a wave of moderate height and period on a mildly sloping beach: $T_p = 8$ s, $m = 0.03$, $\Delta k = 0.2k_0\cos\theta_0$, $\theta_0 = 0$, $\Delta f = 0.2 f_p$, $H_b = 1$ m, $\gamma = 0.78$.

Figure 2 shows how maximum velocities vary with peak period and wave height. Both longshore and cross-shore velocities decrease strongly with increasing period:

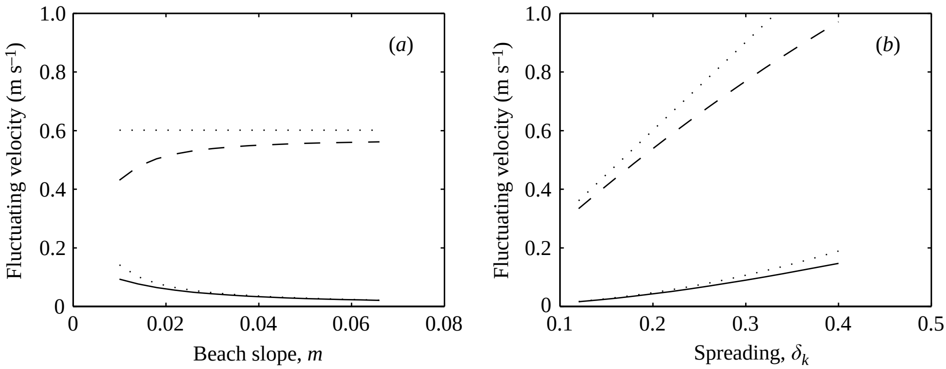

FIGURE 3. As figure 2 but for (*a*) changing beach slope; (*b*) changing directional spreading.

this occurs because of the strong increase in longshore length scales (decrease in Δk). The full relations are seen to agree well with the asymptotic limits for large period, but diverge for short periods as longshore and cross-shore length scales become comparable. The base case, with $T_p = 8$ s, appears on the edge of applicability for the asymptotic relations. Longshore velocities appear significant over a wide range while cross-shore velocities are negligible for large wave periods.

Both cross-shore and longshore velocities increase strongly with increasing wave height, as expected. Again, divergence from the asymptotic limits is seen as larger breaking waves change the aspect ratio of the circulation cell.

Figure 3 predicts a weakly changing longshore velocity with increasing beach slope m, while cross-shore velocities diminish strongly with increasing steepness. Increased directional spreading causes strong increases in fluctuating velocities-again, this suggests that wave spectra with two peak directions but similar frequencies will cause strong 'migrating rip currents' (Fowler & Dalrymple 1990).

4. Mean currents

From Longuet-Higgins (1970*a*, *b*), the maximum longshore current on a longshore-uniform beach scales like

$$\begin{aligned} O(V) &= O\left(\frac{gmh_b}{c_f}\frac{\sin\theta}{C}\right) \\ &= O\left(\frac{gH_bm}{c_f}\frac{f_p\sin\theta_0}{g}\right) \\ &= O\left(H_b f_p \sin\theta_0 \frac{m}{c_f}\right) \end{aligned} \tag{4.1}$$

where c_f is the bottom friction coefficient, θ_0 is the deep water wave angle relative to the shore normal, and C is the wave phase speed. From Snell's law, the ratio $\sin\theta/C$ remains constant, so the deep water dispersion relation was used to simplify the expression.

On a longshore-uniform beach, the ratio of forced rotational fluctuating longshore velocity to mean longshore velocity then becomes

$$O\left(\frac{v}{V}\right) = O\left(\frac{\delta_k}{\delta_f}\frac{c_f}{m}\cot\theta_0\right). \tag{4.2}$$

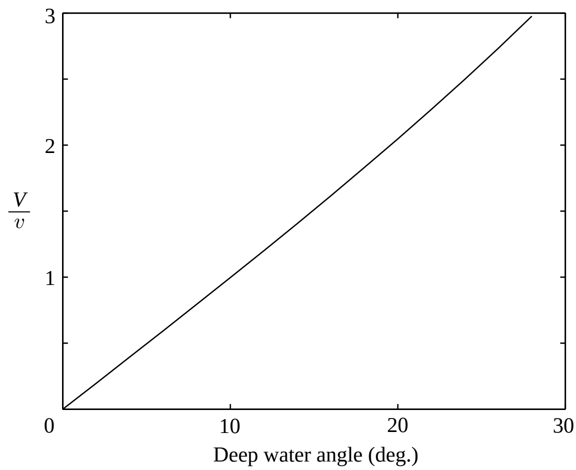

FIGURE 4. Ratio of maximum mean longshore current to amplitude of maximum fluctuating longshore current using base case (see figure 2, and (3.8), (3.16)–(3.17), and (4.3)) with friction coefficient $c_f = 0.005$ and mixing parameter $P = 0.1$.

Thus, the ratio of fluctuating to mean longshore velocity is, perhaps surprisingly, predicted to not depend on wave height or frequency, but instead on details of the wave spectrum, bathymetry and frictional characteristics, and deep water wave angle. The lack of dependence on wave height and period may make it fairly robust for predictive purposes, with the notable complication of the friction factor.

Figure 4 shows the ratio of maximum fluctuating longshore current to maximum mean current for the base case, but with changing deep water wave angle. Quantitative values for the maximum longshore current use $c_f = 0.005$, and mixing coefficient $P = 0.1$ from Longuet-Higgins (1970*b*), so that

$$V_{max} = R\frac{5\pi m H_b}{2c_f} f_p \sin\theta_0 \tag{4.3}$$

where $R = 0.5173$ for $P = 0.1$. This solution neglects differences between still water level and mean water level as they are small, which seems reasonable in this case. For low angles and thus weak longshore currents, the unsteady fluctuations are relatively large, as expected, but mean currents dominate for large deep water angles. This solution is extremely sensitive to the friction factor, which is still not easily predictable *a priori*, so there is considerable uncertainty.

5. Discussion and conclusions

Overall, rotational fluctuating velocities can be significant for some cases, but insignificant in others. On longshore-uniform beaches, fluctuating longshore velocities are significantly larger than those in the cross-shore direction. The location of maximum fluctuating cross-shore and longshore velocities appears sensitive to the detailed distribution of vorticity, but will be somewhere in the surf zone.

Three relatively common situations appear likely to force strong rotational velocity fluctuations:

(*a*) locally generated storm waves,

(*b*) bi-directional spectra, and

(*c*) small-scale ($O(100\,\mathrm{m})$), large-amplitude, longshore topographic variations combined with long-period swell waves.
The common case of swell waves on a longshore-uniform beach is likely to force relatively small rotational fluctuations.

The ratio of fluctuating longshore velocity to mean longshore velocity is found to depend not on wave height or frequency, but on details of the spectrum, mean angle, and bathymetry.

A strong difference is predicted between rotational fluctuations on longshore-uniform and longshore-varying beaches, with short-period wind seas producing larger fluctuations on longshore-uniform beaches, and long-period swell being more effective on longshore-varying beaches.

Causes of errors include neglect of frictional dissipation and simplifications of the rate of generation of circulation. Nonlinear processes would also modify this simple solution, and would almost certainly promote instabilities of these unsteady circulation cells. However, the relative importance of these instabilities compared to shear instabilities of the mean current (Bowen & Holman 1989) remains to be determined.

A. B. K. was supported in this work by the National Oceanographic Partnership Program and by Florida Sea Grant.

REFERENCES

Abramowitz, M. & Stegun, I. A. *Handbook of Mathematical Functions*. Dover.

Bowen, A. J. 1967 Rip currents. PhD thesis, University of California, San Diego.

Bowen, A. J. 1969 Rip currents 1. Theoretical investigations. *J. Geophys. Res.* **74**, 5467–5478.

Bowen, A. J. & Holman, R. A. 1989 Shear instabilities of the mean longshore current 1. Theory. *J. Geophys. Res.* **94**(C12), 18023–18030.

Brocchini, M., Kennedy, A. B., Soldini, L. & Mancinelli, A. 2004 Topographically-controlled, breaking wave-induced macrovortices. Part 1. Widely separated breakwaters. *J. Fluid Mech.* **507**, 289–307.

Bühler, O. & Jacobson, T. E. 2001 Wave-driven currents and vortex dynamics on barred beaches. *J. Fluid Mech.* **449**, 313–339.

Foda, M. A. & Mei, C. C. 1981 Nonlinear excitation of long-trapped wave by a group of short swells. *J. Fluid Mech.* **111**, 319–345.

Fowler, R. E. & Dalrymple, R. A. 1990 Wave group forced nearshore circulation. *Intl Conf. Coastal Engng, ASCE*, pp. 729–742.

Kennedy, A. B. 2003 A circulation description of a rip current neck. *J. Fluid Mech.* **497**, 225–234.

Lascody, L. L. 1998 East central Florida rip current program. *Natl Wea. Dig*. **22**, 25–30.

Longuet-Higgins, M. S. 1970*a* Longshore currents generated by obliquely incident sea waves, 1. *J. Geophys. Res.* **75**, 6778–6789.

Longuet-Higgins, M. S. 1970*b* Longshore currents generated by obliquely incident sea waves, 2. *J. Geophys. Res.* **75**, 6790–6801.

Longuet-Higgins, M. S. & Stewart, R. W. 1962 Radiation stress and mass transport in water waves with application to "surf beats". *J. Fluid Mech.* **8**, 565–583.

Longuet-Higgins, M. S. & Stewart, R. W. 1964 Radiation stresses in water waves; a physical discussion, with applications. *Deep Sea Res.* **11**, 529–562.

Peregrine, D. H. 1998 Surf zone currents. *Theor. Comput. Fluid Dyn.* **10**, 295–309.

Peregrine, D. H. 1999 Large-scale vorticity generation by breakers in shallow and deep water. *Eur. J. Mech.* B **18**, 403–408.